

$x(t) \rightarrow h(t) \rightarrow y(t) = x(t) * h(t)$, $h(t)$ wireless channel
 $h(t) = \sum_{i=0}^{L-1} \alpha_i \delta(t - \tau_i) \rightarrow$ due to scattering
 $y(t) = \text{Re} \left\{ \sum_{i=0}^{L-1} s_b(t - \tau_i) \alpha_i e^{j2\pi f_c \tau_i} \cdot e^{j2\pi f_c t} \right\}$
 complex base band representation of $y(t)$
 Narrow band assumption: $f_m \ll \frac{1}{T_c} \rightarrow s_b(t - \tau_i) \approx s_b(t)$
 $\Rightarrow y_b(t) = s_b(t) \cdot \sum_{i=0}^{L-1} \alpha_i e^{j2\pi f_c \tau_i}$
 fading: variation of power w.r.t time \rightarrow complex fading coefficient (h)
 $h = X + jY = \alpha e^{j\phi}$
 $X, Y \sim \mathcal{N}(0, \sigma^2)$; $f_A(\alpha) = 2\alpha e^{-\alpha^2}$; $\phi \sim \text{unit}(-\pi, \pi)$

- To get knowledge of 'h' it must be measured at least once every coherence time
 - measuring/estimating h is called channel estimation
 - Estimation is done using pilot symbol transmission
 $T_c <$ channel estimation time \leftarrow fast fading
 $T_c >$ channel estimation time \leftarrow slow fading
 generally $\sigma_{\tau} \ll T_c$

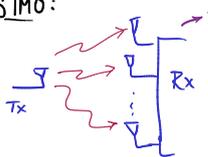
CDMA multiuser downlink:
 $0, 1, \dots, K \rightarrow$ users
 $s_0, s_1, \dots, s_K \rightarrow$ information symbols
 $c_0, c_1, \dots, c_K \rightarrow$ codes of users
 Tx signal: $x(m) = \sum_{i=0}^K s_i c_i(m)$
 Rx signal: $y(m) = \sum_{i=0}^K h_0(i) x(m-i) + n(m)$
 at user 0: $r_0(m) = \frac{1}{N} \sum_{m=0}^N y(m) c_0(m)$
 $\Rightarrow r_0(m) =$ Signal + multipath interference + multi user interference + noise
 $r = \sum_m W^* r(m) \rightarrow$ RAKE receiver
 $\rightarrow \text{SINR} = \frac{N P_0 \|h\|^2}{\sum_{i=0}^K \|h_i\|^2 P_i - P_0 \sum_{i=1}^K |h_i|^4 + \sigma_n^2}$
 for uplink also

wireline | wireless
 SNR: $\frac{P^2}{\sigma_n^2}$ | $\frac{P^2 \alpha^2}{\sigma_n^2}$; α - random variable
 BER = $Q(\sqrt{\text{SNR}})$ | avg. BER = $\frac{1}{2} \left(1 - \sqrt{\frac{\text{SNR}}{\text{SNR} + 2}} \right)$
 $\approx e^{-\text{SNR}}$ | $\approx \frac{1}{2 \text{SNR}}$
 Deep fade \rightarrow noise power > signal power
 $P(\text{deep fade}) \approx \frac{1}{\text{SNR}}$

TDMA \rightarrow [u1] [u2] [u3] used in 2G GSM
 FDMA \rightarrow [u1] [u2] [u3] \rightarrow f used in 1G
 CDMA - used in 3G
 $u_0 \rightarrow a_0 [1 \ 1 \ 1 \ 1]$
 $u_1 \rightarrow a_1 [-1 \ -1 \ -1 \ -1]$
 4 chips $\leftarrow c_1$
 Code = collection of chips
 $N \rightarrow$ # chips in a code; Time duration of chip $(T_c) = \frac{1}{N}$
 spread factor \Rightarrow B.W. $= N \times f \Rightarrow$ spread spectrum
 - # orthogonal codes possible of length N are N

Asynchronous CDMA:
 - happens during uplink

 f - fraction of delay
 $f \sim \text{unit}(0, 1)$, $1-f \sim \text{unit}(0, 1)$
 $r_{01} = f r_0(-1) + (1-f) r_0(0)$
 $\text{SINR} = \frac{\|h_0\|^2 P_0 N}{\sum_{i=0}^K \|h_i\|^2 P_i - \frac{2}{3} P_0 \sum_{d=0}^{L-1} |h_{0d}|^4 + \sigma_n^2}$

SIMO:

 h_i - fading coefficient of link i
 $\bar{y} = \bar{h} x + \bar{n}$; $E[n_i^2] = \sigma_n^2$
 Signal detection: linear combination of y_i 's (Beam forming)
 $\hat{x} = W^T \bar{y}$; $W^* = \frac{\bar{h}}{\|\bar{h}\|} \rightarrow$ Maximal Ratio Combiner (MRC)
 Spatial Matched filter
 $\text{SNR} = \frac{\|\bar{h}\|^2 P}{\sigma_n^2}$
 $\text{avg. BER} \propto \frac{1}{2^L} \cdot \frac{1}{(\text{SNR})^L} \cdot 2^{L-1} C_L \propto \frac{1}{(\text{SNR})^L}$
 $P(\text{deep fade}) \propto \frac{1}{(\text{SNR})^L}$

- codes are generated using Linear Shift Feedback Registers (LSFR)
 - # 1's are 1 more than 0's; correlation = $\begin{cases} 1 & \text{if shift} = 0 \\ -1/N & \text{else} \end{cases}$
 $\frac{1}{2^{L+1}}$ runs of length 2^L
 - Random spread sequence: 1. Each $c_i(i)$ is ± 1 w.p. $1/2$
 2. $c_i(i), c_j(i)$ are independent } $E[c_0(i)] = 0, E[\pi_0(k)] = 0, E[\pi_0(i)] = 0$
 3. $c_i(i), c_j(i)$ are independent } $E[\pi_0^2(k)] = \frac{1}{N}, E[\pi_0^2(k)] = \frac{1}{N}$
 $\pi_{ij}(k) = \frac{1}{N} \sum_m c_i(m) c_j(m+k)$

Near - far problem of CDMA
 $\text{SINR} = \frac{P_0}{\frac{P_1}{N} + \frac{\sigma_n^2}{N}}$
 $P_1 \propto \frac{1}{d^2}$
 Power transmitted by user must be regulated.

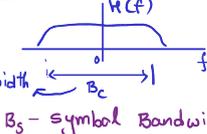
Diversity order = $-\lim_{\text{SNR} \rightarrow \infty} \frac{\log(\text{BER})}{\log(\text{SNR})} \rightarrow L$ SIMO System
 $\rightarrow \infty$ for wireline
 $|h(\tau)|^2 = \sum_{i=0}^{L-1} |\alpha_i|^2 \delta(\tau - \tau_i)$
 max delay spread = $\tau_{L-1} - \tau_0$
 RMS delay spread (σ_{τ})
 $\sigma_{\tau} = \sqrt{\frac{\sum_{i=0}^{L-1} |\alpha_i|^2 (\tau - \bar{\tau})^2}{\sum_{i=0}^{L-1} |\alpha_i|^2}}$; $\bar{\tau} = \frac{\sum_{i=0}^{L-1} |\alpha_i|^2 \tau_i}{\sum_{i=0}^{L-1} |\alpha_i|^2}$
 - avg. Power = $E[|h(\tau)|^2] = \bar{\phi}(\tau)$
 $f(\tau) = \frac{\bar{\phi}(\tau)}{\int_{-\infty}^{\infty} \bar{\phi}(\tau) d\tau}$; $\bar{\tau} = \int_{-\infty}^{\infty} \tau f(\tau) d\tau$; $\sigma_{\tau}^2 = \int_{-\infty}^{\infty} (\tau - \bar{\tau})^2 f(\tau) d\tau$
 Delay spread \approx 1-3 μ s outdoor
 \approx 10-50 ns indoor

Multi user CDMA: $x(n) = a_0 c_0(n) + a_1 c_1(n)$
 at user 0 $y(n) = x(n) + w(n)$
 $r_0 = \frac{1}{N} \sum_n y(n) c_0(n)$
 $= \frac{1}{N} \sum_n a_0 c_0^2(n) + \frac{1}{N} \sum_n a_1 c_0(n) c_1(n) + \frac{1}{N} \sum_n w(n) c_0(n)$
 desired signal | Interference | noise
 $\Rightarrow \text{SNR} = \frac{P_0}{\frac{P_1}{N} + \frac{\sigma_w^2}{N}} = N \left(\frac{P_0}{P_1 + \sigma_w^2} \right)$
 Spread gain

Multiple Input Multiple output (MIMO)
 L - Tx antenna r - Rx antenna
 - spatial multiplexing $r \leq L$
 $\bar{y}_{rx} = H_{rx \times L} \bar{x}_{L \times 1} + \bar{n}_{rx \times 1}$ generally
 - Total rt channel coefficients
 - Noise: $E[n_i^2] = \sigma_n^2, E[n_i n_j^*] = 0$
 Receiver:
 $\bar{y} = H \bar{x} + \bar{n}$; $\hat{x} = H^{-1} \bar{y} - H^{-1} \bar{n}$ (H is not a Square matrix)
 $\hat{x} = (H^T H)^{-1} H^T \bar{y}$ \rightarrow MMSE estimation of Pseudo inverse \rightarrow Zero forcing receiver
 $(r-t+1)$ diversity amplifies noise if h is small ($\frac{r}{t}$ term)

Advantages of CDMA
 - Jammer margin: disruptive user with high power P_I cause interference $\frac{P_I}{N}$ (suppresses Jammer power)
 - Graceful degradation
 $\text{SNR} = \frac{P}{\frac{P_1}{N} + \frac{P_2}{N} + \dots + \frac{\sigma_n^2}{N}}$
 \rightarrow Interference by new user is distributed among other users
 - Universal frequency reuse
 - some frequencies can be used in all cells
 - Interference due to adjacent cell is $\frac{P_I}{N}$
 \rightarrow Multipath diversity
 $y(m) = h(m) x(m) + h(l) x(m-1) + \dots + h(L-1) x(m-L+1)$; $x(m) = s_0 c_0(m)$
 ISI \Rightarrow freq. selective

Linear estimator: $\hat{x} = \bar{E}^T \bar{y}$
 minimize $E[\|\hat{x} - x\|^2]$
 $C^* = R_{yy}^{-1} R_{yx}$ $E[\bar{x} \bar{y}^T] = R_{xy} = R_{yx}^T$
 $= P_y H^T (P_y H^T + \sigma_n^2 I)^{-1} \bar{y}$
 $H^T H \bar{y}$ at low SNR $\frac{P}{\sigma_n^2} H^T \bar{y}$ at high SNR
 \rightarrow (zero forcing) (matched filter)

$H(f) = F(h(t)) \rightarrow$ 
 coherence bandwidth B_c
 $B_c \propto \frac{1}{2\sigma_{\tau}}$ B_s - symbol Bandwidth
 $B_s > B_c \Rightarrow$ frequency selective distortion
 $\sigma_{\tau} \gg T_{\text{symbol}} \Rightarrow$ Inter Symbol Interference (ISI)
 $(2 B_c \ll B_s)$

Change in frequency due to motion \leftarrow doppler effect
 $h(t) = \sum_{i=0}^{L-1} \alpha_i e^{-j2\pi f_c \tau_i} e^{j2\pi f_d t}$
 \rightarrow time varying channel
 Time varying phase $\tau_i(t) = \tau_i - \frac{v \cos \theta_i t}{c}$
 $f_d = f_c \frac{v \cos \theta}{c}$
 mobility \rightarrow doppler \rightarrow Time selective channel.
 coherence Time $T_c = \frac{1}{4 f_d}$ (approx time channel is constant)
 Doppler spread $B_d = 2 f_d$ $T_c \propto \text{ms}$

$r_0(m) = \frac{1}{N} \sum_m y(m) c_0(m) = s_0 h_0(m) + \bar{n}(m)$
 $\bar{n}(m) = \bar{h} s_0 + \bar{n}$ (receiver diversity)
 $r_0(k) = \frac{1}{N} \sum_m y(m) c_0(m-k) = s_0 h_0(k) + \bar{n}(k)$
 - By combining $r = \sum W^* \bar{r} \rightarrow$ RAKE receiver
 for maximizing SNR we get $\bar{W}^* = \frac{\bar{h}}{\|\bar{h}\|}$
 $\text{SNR} = \frac{\|\bar{h}\|^2 P_0}{\sigma_n^2} \cdot N$; $\text{BER} \sim \frac{1}{(\text{SNR})^L}$
 RAKE receiver extracts multipath diversity by coherent combination of multipath components

Singular Value Decomposition (SVD) of H

$H = U \Sigma V^T$; $U U^T = I$, $V V^T = I$
 $\Sigma = \begin{bmatrix} \sigma_1 & & 0 \\ & \sigma_2 & \\ 0 & & \sigma_t \end{bmatrix}$ $\sigma_1 > \sigma_2 > \dots > \sigma_t$
 $\bar{y} = H \bar{x} + \bar{n} = U \Sigma V^T \bar{x} + \bar{n}$
 $\hat{y}_{t \times 1} = U^T \bar{y} = \Sigma V^T \bar{x} + U^T \bar{n} = \Sigma \tilde{x} + \tilde{n}$
 $\sigma_n^2 = E[\tilde{n} \tilde{n}^T] = \sigma_n^2$
 Decoupling/Parallelization of MIMO channels
 \Rightarrow SNR of i^{th} stream = $\frac{P_i \sigma_i^2}{\sigma_n^2}$
 Total capacity = $\sum_{i=1}^t \log_2(1 + \frac{P_i \sigma_i^2}{\sigma_n^2})$

Optimal MIMO power allocation:

$\max_{P_1, \dots, P_t} \sum_{i=1}^t \log_2(1 + \frac{P_i \sigma_i^2}{\sigma_n^2})$
 s.t. $\sum_{i=1}^t P_i \leq P$
 - Water filling algorithm $\frac{1}{\lambda} = ?$
 $P_i \geq P_2 \geq P_3 \dots$
 $P_i = (\frac{1}{\lambda} - \frac{\sigma_n^2}{\sigma_i^2})^+$
 $P = \sum_{i=1}^t (\frac{1}{\lambda} - \frac{\sigma_n^2}{\sigma_i^2})^+$

Asymptotic Capacity:

$C_a = \log_2 |I + \frac{1}{\sigma_n^2} H R_x H^T|$
 $R_x = \frac{P}{t} I$, $t \gg n$, $H H^T \rightarrow t I$
 $\Rightarrow C_a = n \log_2(1 + \frac{P}{\sigma_n^2})$
 $= \min(n, t) \log_2(1 + \frac{P}{\sigma_n^2})$
 $C_a \uparrow$ as $n \uparrow \Rightarrow$ MIMO increases capacity

Alamouti code: \rightarrow for 2 TX, 1 RX system

- Orthogonal Space Time Block Code (OSTBC)
 - Achieves diversity without channel state information (CSI)
 1st Instance $y(1) = [h_1 \ h_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n(1)$
 2nd Instance $y(2) = [h_1 \ h_2] \begin{bmatrix} x_2^* \\ -x_1^* \end{bmatrix} + n(2)$
 At Rx:
 $\begin{bmatrix} y(1) \\ y(2) \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n(1) \\ n(2) \end{bmatrix} \rightarrow 2 \times 2$ MIMO
 $\bar{y}_1^T \bar{y} = \|h\| x_1 + \tilde{n}_1 \Rightarrow SNR = \frac{\|h\|^2 P_1}{\sigma_n^2}$; $w_1 = \frac{c_1}{\|c_1\|}$
 $\bar{y}_2^T \bar{y} = \|h\| x_2 + \tilde{n}_2 \Rightarrow SNR = \frac{\|h\|^2 P_2}{\sigma_n^2}$; $w_2 = \frac{c_2}{\|c_2\|}$
 c_1, c_2 are orthogonal
 $\begin{matrix} x_1 & -x_2^* \\ x_2 & x_1 \end{matrix}$ Transmits 2 symbols / 2 time slots
 \rightarrow Rate = 1 (full rate)

Non linear MIMO receiver

V-BLAST (Vertical Bell Labs space Time)
 - Employs successive Interference cancellation (SIC)
 $\bar{y} = H \bar{x} + \bar{n} = \bar{h}_1 x_1 + \bar{h}_2 x_2 + \dots + \bar{h}_t x_t + \bar{n}$
 $\hat{y}_1 = \bar{y}_1^T \bar{y} = x_1 + \tilde{n}$; $Q = \begin{bmatrix} \bar{y}_1 \\ \bar{y}_2 \\ \vdots \end{bmatrix}$ & $Q H = I$
 $\hat{y}_2 = \bar{y}_2 - \bar{h}_1 x_1 = \bar{h}_2 x_2 + \dots + \bar{h}_t x_t + \tilde{n} \rightarrow$ repeat the process for $x_2 \dots$

Advantages:

- Diversity order progressively increases
- Streams decoded later have higher diversity
- for last symbol n^{th} order diversity.

MIMO Beamforming

- use directional antennas
 $\bar{y} = U \Sigma V^T \bar{x} + \bar{n}$; $V V^T = I$
 $\bar{x} = V \tilde{x} \rightarrow$ (MRT)
 Maximal Ratio Transmission
 $\bar{y} = \sigma_1 \tilde{x}_1 \bar{u}_1 + \bar{n}$
 $\bar{u}_1^T \bar{y} = \sigma_1 \tilde{x}_1 + \bar{n}_1 \Rightarrow SNR = \frac{\sigma_1^2 P}{\sigma_n^2}$
 - MRT is capacity optimal at low SNR
 - Achieves complete diversity order
Advantage:
 Simple Tx and Rx scheme for MIMO

B - Bandwidth available for communication

$T = \frac{1}{B} \rightarrow$ symbol duration
 If we tx 1 symbol for every T sec
 Then symbol rate = $\frac{1}{T} = B$
Multi carrier system: N - subcarriers
 $f_i = \frac{iB}{N}$
 $x_i \rightarrow$ data transmitted on i^{th} subcarrier

Tx signal: $S(t) = \sum_{i=1}^N x_i e^{j2\pi i \frac{B}{N} t}$
 $S_i(t) \rightarrow$ duration $\frac{N}{B}$

If no noise
 Rx signal: $y(t) = \sum_{i=1}^N x_i e^{j2\pi i \frac{B}{N} t}$
 By coherent demodulation $\hat{x}_i = \int y(t) e^{-j2\pi i \frac{B}{N} t} dt$

- This scheme is called Multi carrier Modulation (MCM)
- Symbol rate = $\frac{N}{N/B} = B$ (same as single carrier)
- $B = 1024$ kHz \rightarrow Single carrier $\rightarrow B > B_c$ ($\sim 200-300$ kHz) \rightarrow (ISI, freq. selective)
- \rightarrow Multicarrier $\rightarrow \frac{B}{N} \ll B_c \Rightarrow$ Flat fading (no ISI)

Bottle neck:

- Implementing N modulators and demodulators

Orthogonal Frequency Division Multiplexing (OFDM)

- Sample $s(t)$ with $T_s = \frac{1}{B}$
 u^{th} sample: $s(uT_s) = \sum_{i=0}^{N-1} x_i e^{j2\pi i u \frac{1}{N}}$
 IDFT of $[x(0) x(1) \dots x(N-1)]$
 - Implementing this have lower complexity than correlator \rightarrow Prev symbol
 $y(t) = h(t) x(t) + h(t) \tilde{x}(N-1) + \dots + h(t-N) \tilde{x}(N-L+1)$
cyclic prefix: To avoid ISI and make channel multiple flat fading channels
By cyclic prefix: \rightarrow has only samples of $x \rightarrow$ circular convol.
 $y(t) = h(t) x(t) + h(t) x(t-N) + \dots + h(t-L) x(t-L+1)$
 $\Rightarrow Y(k) = H(k) \cdot X(k) \rightarrow$ Flat fading for k^{th} subcarrier

wide band Frequency became group of narrowband selective channel

Sample detection: By Zero forcing $\hat{x}(k) = \frac{1}{H(k)} Y(k)$
 By matched filter: $H^*(k) Y(k) = |H(k)|^2 x(k) + N'(k)$
 By MMSE: $\hat{x}_{MMSE}(k) = \frac{H^*(k)}{|H(k)|^2 + \sigma_n^2} Y(k)$

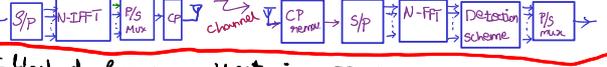
- Max length of cyclic prefix = L-1
 \Rightarrow Loss of efficiency = $\frac{L-1}{N+L-1} \xrightarrow{N \rightarrow \infty} 0$

$N T \rightarrow$ symbol duration $\uparrow \rightarrow$ increases decoding time
 Intuitively: $N T_s \geq T_d \Rightarrow N_c \geq \frac{1}{2} \frac{B}{B_c}$ Standard 12.5% symbol time
 $N \gg N_c \gg \frac{B}{B_c} \Rightarrow B_c \gg \frac{B}{N}$ for flat fading

MIMO-OFDM Freq. selective MIMO channel modelled as

$\bar{y}_{n \times 1}(k) = \sum_{l=0}^{L-1} H(k) \bar{x}(k-l) + \bar{n}(k)$
 We need to perform IFFT at each Tx antenna.
 $\bar{y}(0) = \tilde{H}(0) \bar{x}(0)$ $\tilde{H}(0), \tilde{H}(1), \dots, \tilde{H}(k) \rightarrow$ flat fading channels
 $\hat{y}(k) = \tilde{H}(k) \bar{x}(k) \rightarrow$ FFT of $H(k)$
 - Each $\bar{y}(0), \dots, \bar{y}(N-1)$ can be processed by simple MIMO Z forcing (or) MIMO MMSE receiver for detection.
 $\hat{x}(k) = (\tilde{H}(k))^T \hat{y}(k)$; $\hat{x}(k) = P (\tilde{H}^T \tilde{H} + \sigma_n^2 I)^{-1} \tilde{H}^T \hat{y}(k)$

Schematic of OFDM with cyclic Prefix



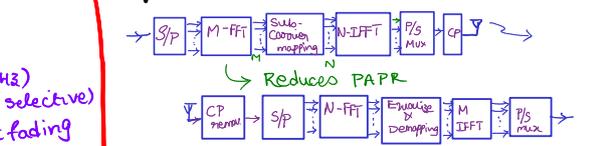
Effect of frequency offset in OFDM:

- This leads to loss of orthogonality among subcarriers thus causing Inter carrier Interference (ICI)
 let $\Delta f =$ freq. offset, $\epsilon = \frac{\Delta f}{B/N} \rightarrow$ normalized freq. offset
 L^{th} coefficient if $\epsilon \neq 0$
 $Y_L = \frac{1}{N} \sum_n y_n e^{j2\pi n \epsilon}$
 $= \frac{1}{N} \sum_n x_k H_k e^{j2\pi n \epsilon / N} + \frac{1}{N} \sum_{k \neq L} x_k H_k e^{j2\pi n (k-L) \epsilon / N} + \tilde{w}_L$
ICI
 $SINR = \frac{P |H_L|^2 (\text{sinc} \epsilon)^2}{0.6922 P |H_L|^2 (\text{sinc} \epsilon)^2 + \sigma_n^2}$ for $N \rightarrow \infty$

Soft handover \rightarrow change of code in CDMA

Hard handover \rightarrow change in frequency

Single Carrier FDM access (SCFDMA):



Sub carrier mapping: $M < N$

1. Interleave $x(1), 0, 0, 0, x(2), 0, 0, 0, x(3), \dots$
2. Zero padding \rightarrow SFDMA $x(1), x(2), x(3), 0, 0, \dots$

Wireless channel modelling:

- large scale \rightarrow mean signal strength propagation models at Rx
- a) Free space model $P_r(d) = \frac{P_t G_t G_r \lambda^2}{4\pi d^2 L}$; $P_{\alpha} \frac{1}{d^\alpha}$ Path loss exponent
 - b) Ground reflection $\rightarrow P_{\alpha} \frac{1}{d^\alpha}$
 - c) Okumura model - more practical, widely used in urban - Predicts median loss - valid range 1050-1920 MHz, can be extrapolated
 - d) Hata model - another popular model for urban
 - e) Lognormal shadowing

Link budget:

+	Transmit power	P_t
+	Gain	G_t
-	Median loss propagation	L_{50}
-	Margin	M_{dB}
+	Mobile Rx Gain	G_r
-	cable loss	L_n
-	Receiver (noise + interference) $N+I$	
=	Required SNR	SNR_{req}