$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t) = x(t) * h(t)$, $h(t)$ wireless channel

$h(t) = \sum_{i=0}^{L-1} a_i \, \delta(t-\tau_i)$ → due to scattering

$y(t) = Re\{\underbrace{\sum_{i=0}^{L-1} s_b(t-\tau_i)\, a_i e^{j2\pi f_c \tau_i}}_{\text{complex base band representation of } y(t)} \cdot e^{j2\pi f_c t}\}$

Narrow band assumption: $f_m < \frac{1}{\tau_i} \Rightarrow s_b(t-\tau_i) \approx s_b(t)$

$\Rightarrow y_b(t) = s_b(t) \cdot \underbrace{\sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i}}$ → complex fading coefficient (h)

fading: variation of power w.r.t time

$h = X + jY = a\, e^{j\phi}$

$X, Y \sim \mathcal{N}(0, 1/2)$; $f_A(a) = 2a\, e^{-a^2}$; $\phi \sim$ unif $(-\pi, \pi)$

- To get knowledge of 'h' it must be measured atleast once every coherence time
- measuring/estimating h is called channel estimation
- Estimation is done using pilot symbol transmission

$T_c <$ channel estimation time ← fast fading

$T_c >$ channel estimation time ← slow fading

generally $\sigma_\tau << T_c$

T_c ↑

flat fading slow fading	Freq. selective fading slow fading
flat fading fast fading	Freq. selective fading fast fading

→ σ_τ

CDMA multiuser down link:

$0, 1, \ldots K$ → users

$s_0, s_1 \ldots s_K$ → Information symbols

$c_0, c_1 \ldots c_K$ → codes of users

Tx signal: $x(m) = \sum_{i=0}^{K} s_i c_i(m)$

Rx signal: $y(m) = \sum_{i=0}^{L-1} h_0(i) x(m-i) + n(m)$

at user 0: $r(0) = \frac{1}{N} \sum_{m=0}^{N} y(m) c_0(m)$

$\Rightarrow r(0) =$ Signal + Multi path interfence + Multi user interference + noise

$r = \sum_m w^* r(m)$ → Rake reciver

$\rightarrow SINR = \frac{N P_0 \|h_0\|^2}{\sum_{i=0}^{K} \|h_0\|^2 P_i - P_0 \sum_{i=1}^{L-1} \frac{|h_i|^4}{\|h_0\|^2} + \sigma_n^2}$

↙ for uplink also

	wireline	wireless
SNR:	$\frac{p2}{\sigma_n^2}$	$\frac{p^2 a^2}{\sigma_n^2}$; a - random variable
	BER $= Q(\sqrt{SNR})$ $\approx e^{-SNR}$	avg. BER $= \frac{1}{2}\left(1 - \sqrt{\frac{SNR}{SNR+2}}\right)$

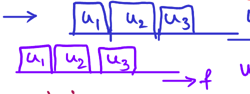
$\approx \frac{1}{2\,SNR}$

Deep fade → Noise power > signal power

$\mathbb{P}(\text{deep fade}) \approx \frac{1}{SNR}$

TDMA →

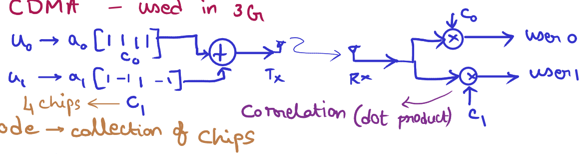
FDMA →



used in 2G GSM

used in 1G

CDMA - used in 3G



Code → collection of chips

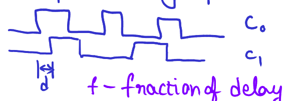
N → # chips in a code (or) spread factor ; Time duration of chip (T_c) $= \frac{T}{N}$

$\Rightarrow$ B.W $= N \times f$ → spread spectrum

- # orthogonal codes possible of length N are N

Asynchronous CDMA:

- happens during uplink



f - fraction of delay

$f \sim$ unif$(0,1)$, $1-f \sim$ unif$(0,1)$

$r_{01}^a = f\, r_{01}(-1) + (1-f)\, r_{01}(0)$

$SINR = \frac{\|h_0\|^2 P_0 N}{\frac{2}{3}\sum_{i=0}^{K} \|h_i\|^2 P_k - \frac{2}{3} P_0 \sum_{d=0}^{L-1} \frac{|h_0(d)|^4}{\|h\|^2} + \sigma_n^2}$

- Codes are generated using Linear Shift Feedback Registers (LSFR)
  - # 1s are 1 more than 0's ; correlation (dot product) $= \begin{cases} 1 & \text{if shift} = 0 \\ -\frac{1}{N} & \text{else} \end{cases}$
  - $\frac{1}{2^{i+1}}$ runs of length 2^i
- Random spread sequence: 1. Each c(i) is ±1 w.p 1/2
  2. c(i), c(j) are independent
  3. $c_0(i)$, $c_1(i)$ are independent

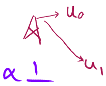
$\mathbb{E}[c_0(i)] = 0$, $\mathbb{E}[r_{00}(K)] = 0$, $\mathbb{E}[r_{01}(K) = 0]$

$\mathbb{E}[r_{00}^2(K)] = \frac{1}{N}$, $\mathbb{E}[r_{01}^2(K)] = \frac{1}{N}$

$r_{ij}(K) = \frac{1}{N} \sum_m c_i(m) c_j(m+K)$

Near - far problem of CDMA

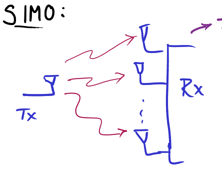
$SINR = \frac{P_0}{\frac{P_1}{N} + \frac{\sigma_n^2}{N}}$



$P_r \propto \frac{1}{d^2}$

Power transmitted by user must be regulated.

SIMO:



→ Improves BER

h_i - fading coefficient of link i

$\bar{y} = \bar{h} x + \bar{n}_{L\times1}$; $\mathbb{E}[|n_i^2|] = \sigma_n^2$

Signal detection: linear combination of y_i's (Beam forming)

$\hat{x} = w^{*T} \bar{y}$; $w^* = \frac{\bar{h}}{\|\bar{h}\|}$ → Maximal Ratio Combiner (MRC) (or) Spacial Matched filter

$SNR = \frac{\|h\|^2 . P}{\sigma_n^2}$

avg. BER $\approx \frac{1}{2^L} \cdot \frac{1}{(SNR)^L} \cdot {}^{2L-1}C_L \propto \frac{1}{(SNR)^L}$

$\mathbb{P}(\text{deep fade}) \propto \frac{1}{(SNR)^L}$

Diversity order $= -\lim_{SNR\to\infty} \frac{\log(BER)}{\log(SNR)}$ → L SIMO System; → ∞ for wireline

Multi user CDMA:

$x(n) = a_0 c_0(n) + a_1 c_1(n)$

$y(n) = x(n) + w(n)$

at user 0

$r_0 = \frac{1}{N} \sum_n y(n) c_0(n)$

$= \underbrace{\frac{1}{N} \sum_n a_0 c_0^2(n)}_{\text{desired signal}} + \underbrace{\frac{1}{N} \sum_n a_1 c_0(n) c_1(n)}_{\text{Interference}} + \underbrace{\frac{1}{N} \sum w(n) c_0(n)}_{\text{noise}}$

↙ $(r_{01}(0))$

$\Rightarrow SNR = \frac{P_0}{\frac{P_1}{N} + \frac{\sigma_w^2}{N}} = N\left(\frac{P_0}{P_1 + \sigma_w^2}\right)$

↙ Spread gain

Multiple Input Multiple output (MIMO)

t - Tx antenna r - Rx antenna

- spatial multiplexing

$\bar{y}_{r\times1} = H_{r\times t} \bar{x}_{t\times1} + \bar{n}_{r\times1}$ $r \le t$ generally

- Total rt channel coefficients
- Noise: $\mathbb{E}[|n_i|^2] = \sigma_n^2$, $\mathbb{E}[n_i n_j^*] = 0$

Receiver:

$\bar{y} = H\bar{x} + \bar{n}$; $\hat{x} = H^{-1}y - H^{-1}n$ (H is not a square matrix)

$\Rightarrow \hat{x} = \underbrace{(H^T H)^{-1} H^T}_{\text{Pseudo inverse}} \bar{y}$ → MMSE estimator of $\|y - Hx\|^2$ → Zero forcing receiver

↙ (r-t+1) diversity

↓ amplifies noise if h is small ($\frac{n}{h}$ term)

Linear estimator. $\hat{x} = \bar{c}^T \bar{y}$

minimize$_{\bar{c}}$ $E[\|\hat{x} - x\|^2]$

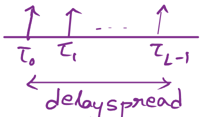
$c^* = R_{yy}^{-1} R_{yx}$ $\mathbb{E}[\bar{x}\bar{y}^T] = R_{xy} = R_{yx}^T$

$= P_d H^T (P_d H H^T + \sigma_n^2 I)^{-1} \bar{y}$

↙ $(H^T H)^{-1} H \bar{y}$ at low SNR ↓ (zero forcing)

↳ at high SNR $\frac{P_d}{\sigma_n^2} H^T \bar{y}$ (matched filter)

$|h(\tau)|^2 = \sum_{i=0}^{L-1} |a_i|^2 \delta(\tau - \tau_i)$



max delay spread $= \tau_{L-1} - \tau_0$

RMS delay spread (σ_τ)

$\sigma_\tau = \sqrt{\frac{\sum_{i=0}^{L-1} |a_i|^2 (\tau - \bar{\tau}_i)^2}{\sum_i |a_i|^2}}$; $\bar{\tau} = \frac{\sum_i |a_i|^2 \tau_i}{\sum_i |a_i|^2}$

↗ weighted power

↳ Total power

- avg. Power $= \mathbb{E}[|h(\tau)|^2] = \bar{\phi}(\tau)$

$f(\tau) = \frac{\bar{\phi}(\tau)}{\int_0^\infty \bar{\phi}(\tau) d\tau}$; $\bar{\tau} = \int_0^\infty \tau f(\tau) d\tau$; $\sigma_\tau^2 = \int_0^\infty (\tau - \bar{\tau})^2 f(\tau) d\tau$

Delay spread ≈ 1-3 μs outdoor

≈ 10-50 ns indoor

Advantages of CDMA

→ Jammer margin

disruptive user with high power P_I cause interference $\frac{P_I}{N}$ (supress Jammer power)

→ Graceful degradation

$SNR = \frac{P_0}{\frac{P_1}{N} + \frac{P_2}{N} + \cdots + \frac{\sigma_n^2}{N}}$

⇒ Interference by new user is distributed among other users

→ universal frequency reuse

- Same frequencies can be used in all cells
- Interfence due to adjacent cell is $\frac{P_1}{N}$

→ Multipath diversity

$y(m) = h(0) x(m) + \underbrace{h(1) x(m-1) + \cdots + h(L-1) x(m-L+1)}_{\text{ISI} \Rightarrow \text{freq. selective}}$; $x(m) = s_0 c_0(m)$

$r(0) = \frac{1}{N} \sum_m y(m) c_0(m) = s_0 h(0) + \tilde{n}(0)$

⋮

$r(K) = \frac{1}{N} \sum_m y(m) c_0(m-K) = s_0 h(K) + \tilde{n}(K)$

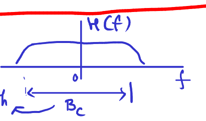
$\Rightarrow \bar{r} = \bar{h} s_0 + \bar{n}$ (receiver diversity)

- By combining $r = \sum \bar{w}^* . \bar{r}$ → RAKE reciver for maximizing SNR we get $\bar{w}^* = \frac{\bar{h}}{\|\bar{h}\|}$

$SNR = \frac{\|h\|^2 P_0}{\sigma_n^2} . N$; $BER \sim \frac{1}{(SNR)^L}$

RAKE reciver extracts multipath diversity by coherent combination of multipath components

$H(f) = \mathcal{F}(h(t))$ →



coherence bandwidth ← B_c

$B_c \approx \frac{1}{2\sigma_\tau}$ B_s - symbol Bandwidth

$B_s > B_c \Rightarrow$ frequency selective distortion

⇕

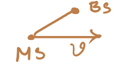
$\sigma_\tau >> T_{symbol} \Rightarrow$ Inter symbol Interference (ISI)

$(2B_c << B_s)$

Change in frequency due to motion ← Doppler effect

$h_b(t) = \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i} \underbrace{e^{j2\pi f_d t}}_{\text{time varying phase}}$

↳ time varying channel



$\tau_i(t) = \tau_i - \frac{v \cos\theta}{c} t$

$f_d = f_c \frac{v \cos\theta}{c}$

mobility → doppler → Time selective channel.

Coherence Time $T_c = \frac{1}{4 f_d}$ (approx time channel is constant)

Doppler spread $B_d = 2 f_d$ $T_c \approx$ ms

Singular Value Decomposition (SVD) of H

$$H = U\Sigma V^T;\quad UU^T = I,\quad VV^T = I$$

$$\Sigma = \begin{bmatrix}\sigma_1 & & 0\\ & \sigma_2 & \\ 0 & & \ddots\ \sigma_t\end{bmatrix}\qquad \sigma_1 > \sigma_2 > \dots > \sigma_t$$

$$\bar{y} = H\bar{x} + \bar{n} = U\Sigma V^T\bar{x} + \bar{n}$$

$$\Rightarrow \underset{t\times1}{\tilde{y}} = U^T\bar{y} = \Sigma \underbrace{V^T x}_{\tilde{x}} + U^T\bar{n} = \overbrace{\Sigma\tilde{x} + \tilde{n}}$$

Decoupling/Parallelization of MIMO channels

$$\sigma_{\tilde{n}}^2 = E[\tilde{n}\tilde{n}^T] = \sigma_n^2$$

$\Rightarrow$ SNR of i^{th} stream $= \dfrac{P_i\sigma_i^2}{\sigma_n^2}$

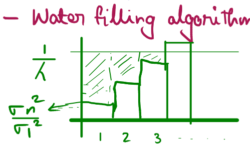
Total capacity $= \sum_{i=1}^{t}\log_2\left(1 + \dfrac{P_i\sigma_i^2}{\sigma_n^2}\right)$

Optimal MIMO power allocation:

$$\max_{P_1\dots P_t}\sum_{i=1}^{t}\log\left(1 + \frac{P_i\sigma_i^2}{\sigma_n^2}\right)$$

$$\text{s.t.}\quad \sum_{i=1}^{t}P_i \le P$$

- Water filling algorithm $\frac{1}{\lambda} = ?$



$P_1 \ge P_2 \ge P_3 \dots$

$$P_i = \left(\frac{1}{\lambda} - \frac{\sigma_n^2}{\sigma_i^2}\right)^+$$

$$P = \sum_{i=1}^{t}\left(\frac{1}{\lambda} - \frac{\sigma_n^2}{\sigma_i^2}\right)^+$$

Asymptotic capacity:

$$C_a = \log_2\left|I + \frac{1}{\sigma_n^2}HR_xH^T\right|;$$

$$R_x = \frac{P_t}{t},\quad t >> r,\quad HH^T \to tI$$

$$\Rightarrow C_a = r\log_2\left(1 + \frac{P_t}{\sigma_n^2}\right)$$

$$= \min(r,t)\log_2\left(1 + \frac{P_t}{\sigma_n^2}\right)$$

$C_a\uparrow$ as $r\uparrow$ $\Rightarrow$ MIMO increases capacity

Alamouti code: → for 2Tx, 1Rx system

- orthogonal space Time Block code (OSTBC)
- Achives diversity without channel state Information (CSI)

1^{st} Instance

$$y(1) = [h_1\ \ h_2]\begin{bmatrix}x_1\\ x_2\end{bmatrix} + n(1)$$

2^{nd} Instance

$$y(2) = [h_1\ \ h_2]\begin{bmatrix}-x_2^*\\ x_1\end{bmatrix} + n(2)$$

At Rx:

$$\begin{bmatrix}y(1)\\ y^*(2)\end{bmatrix} = \begin{array}{c}\begin{bmatrix}h_1 & h_2\\ h_2^* & -h_1^*\end{bmatrix}\\ \begin{array}{cc}c_1 & c_2\end{array}\end{array}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}n(1)\\ n^*(2)\end{bmatrix} \to 2\times2\ \text{MIMO}$$

$$\bar{w}_1^T\bar{y} = \|h\|x_1 + \tilde{n}_1 \Rightarrow SNR = \frac{\|h\|^2P_1}{\sigma_n^2};\quad w_1 = \frac{c_1}{\|c_1\|}$$

$$\bar{w}_2^T\bar{y} = \|h\|x_2 + \tilde{n}_2 \Rightarrow SNR = \frac{\|h\|^2P_2}{\sigma_n^2};\quad w_2 = \frac{c_2}{\|c_2\|}$$

c_1, c_2 are orthogonal

$$\text{space}\downarrow\ \begin{matrix}x_1 & -x_2^*\\ x_2 & x_1\end{matrix}\qquad \xrightarrow{\ \ }\text{time}$$

Transmits 2symbols/2 time slots

⇒ Rate = 1 (full rate)

Non linear MIMO reciver

V-BLAST (vertical Bell Labs space Time)

- Employs successive Interference cancellation (SIC)

$$\bar{y} = H\bar{x} + \bar{n} = \bar{h}_1x_1 + \bar{h}_2x_2 + \dots + \bar{h}_tx_t + \bar{n}$$

$$\tilde{y}_1 = \bar{q}_1^T\bar{y} = x_1 + \tilde{n};\quad Q = \begin{bmatrix}\bar{q}_1\\ \bar{q}_2\\ \vdots\\ \bar{q}_t\end{bmatrix}\ \&\ QH = I$$

$$\hat{y}_2 = \bar{y} - \bar{h}_1x_1 = \bar{h}_2x_2 + \dots + \bar{h}_tx_t + \bar{n} \to \underset{\text{MIMO}}{r\times t-1}$$

— repeat the process for x_2 . . . .

Advantages:

- Diversity order progressively increases
- Streams decoded later have higherdiversity
  - for last symbol r^{th} order diversity.

MIMO Beam forming

- use directional antennas

$$\bar{y} = U\Sigma V^T\bar{x} + \bar{n};\quad V^Tv_1 = \begin{bmatrix}1\\ 0\\ 0\\ \vdots\\ 0\end{bmatrix}$$

$$\bar{x} = v_1\tilde{x}_1 \to (\text{MRT})$$

Maximal Ratio Transmission

$$\bar{y} = \sigma_1\tilde{x}_1\bar{u}_1 + \bar{n}$$

↳ dominant direction

$$\bar{u}_1^T\bar{y} = \sigma_1\tilde{x}_1 + \tilde{n}_1 \Rightarrow SNR = \frac{\sigma_1^2P}{\sigma_n^2}$$

- MRT is capacity optimal at low SNR
- Achives complete diversity order

Advantage:

Simple Tx and Rx scheme for MIMO

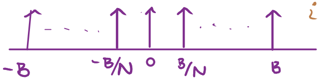
B - Bandwidth available for communication

$T = \frac{1}{B}$ → symbol duration

If we tx 1 symbol for every T sec Then symbol rate $= 1/T = B$

Multi carrier system:

N - subcarriers



i^{th} subcarrier $f_i = \frac{iB}{N}$

X_i → data transmitted on i^{th} sub carrier

Tx signal: $S(t) = \sum_{i=1}^{N}\underbrace{X_i\,e^{j2\pi i\frac{B}{N}t}}_{S_i(t)}$ → duration $\frac{N}{B}$

If no noise

Rx signal: $y(t) = \sum_{i=1}^{N}X_i\,e^{j2\pi i\frac{B}{N}t}$

- By coherent demodulation $\hat{x}_i = \frac{B}{N}\int y(t)\cdot e^{j2\pi i\frac{B}{N}t}\,dt = x_i$ → duration of detection

This scheme is called Multi carrier modulation (MCM)

- Symbol rate $= \dfrac{N}{N/B} = B$ (same as single carrier)

B=1024 KHz → Single carrier → $B > B_c$ ($\sim$200-300 KHz) ↳(ISI, freq. selective)

→ Multicarrier → $\frac{B}{N} << B_c \Rightarrow$ Flat fading (NO ISI)

Bottle neck:

- Implementing N modulators and N-demods.

Orthogonal Frequency Division Multiplexing (OFDM)

- Sample s(t) with $T_s = \frac{1}{B}$

u^{th} Sample: $s(uT_s) = \underbrace{\sum_{i=0}^{N-1}X_i\,e^{j\frac{2\pi}{N}iu}}$

IDFT of $[x(0)\,x(1)\dots x(N-1)]$

- Implementing this have lower complexity than Correlators

$$y(0) = h(0)x(0) + h(1)\tilde{x}(N-1) + \dots + h(N-1)\tilde{x}(N-L+1)$$

→ Prev symbol

Cyclic prefix: To avoid ISI and make channel multiple flat fading channels

By cyclic prefix: → has only samples of x   circular convol.

$$y(0) = h(0)x(0) + h(1)x(N-1) + \dots + h(L-1)x(N-L+1)$$

$\Rightarrow Y(K) = H(K)\cdot X(K)$ → Flat fading for K^{th} sub carrier

∴ wide band Frequency selective channel became group of narrow band flat fading channels

Sample detection: By Zero forcing $\hat{X}(K) = \frac{1}{H(K)}Y(K)$

By matched filter: $H^*(K)Y(K) = |H(K)|^2X(K) + N'(K)$

By MMSE: $\hat{X}_{MMSE}(K) = \dfrac{H^*(K)}{|H(K)|^2 + \sigma_n^2}Y(K)$

- Max length of cyclic prefix = L-1

⇒ Loss of efficiency $= \dfrac{L-1}{N+L-1} \xrightarrow{N\to\infty} 0$

$N\uparrow \Rightarrow$ symbol duration$\uparrow \Rightarrow$ increases decoding time

Intutively: $N_{cp}T_s \ge T_d \Rightarrow N_{cp} \ge \frac{1}{2}\frac{B}{B_c}$ Standard

12.5% symbol time

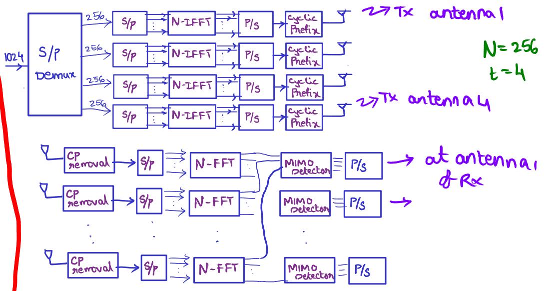
$N >> N_{cp} >> \frac{B}{B_c} \Rightarrow B_c >> \frac{B}{N}$ for flat fading

MIMO-OFDM

Frequency selective MIMO channel modelled as

$$\underset{r\times1}{\bar{y}}(K) = \sum_{l=0}^{L-1}H(l)\bar{x}(K-l) + \bar{n}(K)$$

- we need to perform IFFT at each Tx antenna



$$\bar{Y}(0) = \tilde{H}(0)\bar{X}(0)\qquad \tilde{H}(0)\,\tilde{H}(1)\dots\tilde{H}(K) - \text{flat fading}$$

$$\bar{Y}(K) = \tilde{H}(K)\bar{X}(K)$$

↳ t×1 vector

$$\hat{X}(K) = \tilde{H}(K)^{\dagger}\bar{y}(K),\quad \hat{x}_{MMSE}(K) = P_d(P_d\tilde{H}^T(K)\tilde{H}(K) + \sigma^2I)\tilde{H}^T\bar{y}(K$$

Effect of frequency offset in OFDM:

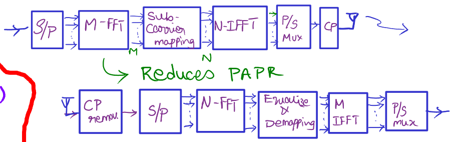
- This cause loss of orthogonality among sub carrier and Inter carrier Interference (ICI)
- Δf - freq. offset, normalized offset $(\varepsilon) = \dfrac{\Delta f}{B/N}$

If $\varepsilon \ne 0$ then l^{th} coefficient

$$Y_l = \underbrace{\frac{1}{N}\sum_n X_KH_K\,e^{j2\pi\varepsilon/N}}_{\text{signal}} + \underbrace{\frac{1}{N}\sum_n\sum_{K=-N/2}^{N/2}X_KH_K\,e^{j2\pi\frac{n}{N}(K-l+\varepsilon)}}_{\text{ICI}} + \underbrace{\tilde{N}_l}_{\text{noise}}$$

$$SINR = \frac{P|H|^2\left(\frac{\sin(\pi\varepsilon)}{\pi\varepsilon}\right)^2}{0.822P|H|^2(\sin\pi\varepsilon)^2 + \sigma_n^2}\ \text{as}\ N\to\infty$$

Single Carrier FDM access (SCFDMA):



Sub carrier mapping: M < N

1. Interleave

   X(1),0,0,0,X(2),0,0,0,X(3) . . .

2. Zero padding → SFDMA

   X(1), X(2), X(3), 0, 0 . . .

Wireless channel modelling:

Large scale propagation models → mean signal strength at Rx

a) Free space model

$$P_r(d) = \frac{P_tG_tG_r\lambda^2}{4\pi d^2L};\quad P\propto\frac{1}{d^2}$$

Path loss exponent

b) Ground reflection → $P\propto\frac{1}{d^4}$

c) Okumura model

- more practical, widely used in urban
- Predicts median loss
- valid range 1050-1920 MHz, can be extrapolated

d) Hata model

- another popular model for urban

e) Lognormal shadowing

Link budget:

+	Transmit power	P_t
+	Gain	G_t
-	Median loss propagation	L^{50}
-	Margin	M_{dB}
+	Mobile Rx Gain	G_r
-	cable loss	L_r
-	Reciver (noise + Interference)	N+I
=	Required SNR	SNR_{req}

Soft handover → change code

hard handoff → change freq.